# EE 330 Lecture 43

#### **Digital Circuits**

- Logic Effort
- Elmore Delay
- Power Dissipation

# EE 330 Lecture 42

#### **Digital Circuits**

- Propagation Delay with Arbitrary Gate Sizing
- Optimally driving large capacitive loads
- Logic Effort

# **Exam Schedule**

Exam 1Friday Sept 24Exam 2Friday Oct 22Exam 3Friday Nov 19FinalTues Dec 14 12:00 p.m.



As a courtesy to fellow classmates, TAs, and the instructor

Wearing of masks during lectures and in the laboratories for this course would be appreciated irrespective of vaccination status

## Summary: Propagation Delay in Multiple-Levels of Logic with Stage Loading

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	Equal Rise/Fall	Equal Rise/Fall (with OD)	Minimum Sized	
$C_{\text{IN}}/C_{\text{REF}}$				
Inverter	1	OD	1/2	
NOR	$\frac{3k+1}{4}$	$\frac{3k+1}{4} \bullet OD$	1/2	
NAND	$\frac{3+k}{4}$	$\frac{3+k}{4} \bullet OD$	1/2	
Overdrive				
Inverter HL	1	OD	1	
LH NOR HL	1	OD	1/3	
	1	OD	1	
LH NAND HL	1	OD	1/(3k)	
	1	OD	1/k	
LH	1	OD	1/3	

\_ k∙

#### **Review from Last Time**

## **Optimal Driving of Capacitive Loads**



Order reduction strategy : Assume overdrive of stages increases by the same factor clear until the load



This becomes a 2-parameter optimization (minimization) problem ! Unknown parameters:  $\{\theta, n\}$ 

**One constraint** :  $\theta^{n}C_{REF} = C_{L}$ 

One degree of freedom

**Review from Last Time** 

### **Optimal Driving of Capacitive Loads**







$t_{PROP} = t_{REF} \frac{\theta}{\ln(\theta)} \left[ \ln \frac{C_{L}}{C_{REF}} \right]$	$t_{PROP} = t_{REF} e \left[ ln \frac{C_{L}}{C_{REF}} \right] = n \theta t_{REF}$
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#### **Review from Last Time**

## **Optimal Driving of Capacitive Loads**



Example: Design a pad driver for driving a load capacitance of 10pF with equal rise/fall times, determine  $t_{PROP}$  for the pad driver, and compare this with the propagation delay for driving the pad with a minimum-sized reference inverter.

In 0.5u proc t<sub>REF</sub>=20ps, C<sub>REF</sub>=4fF,R<sub>PDREF</sub>=2.5K

For 
$$\theta = 2.5$$
, n=8 W<sub>REF</sub>=W<sub>MIN</sub>  
W<sub>nk</sub>=2.5<sup>k-1</sup>•W<sub>REF</sub>, W<sub>pk</sub>=3•2.5<sup>k-1</sup>•W<sub>REF</sub>

<b>–</b> п––р––мім							
k	n-channel		p-channel				
1	1	WMIN		3	WMIN		
2	2.5	WMIN		7.5	WMIN		
3	6.25	WMIN		18.75	WMIN		
4	15.6	WMIN		46.9	WMIN		
5	39.1	WMIN		117.2	WMIN		
6	97.7	WMIN		293.0	WMIN		
7	244.1	WMIN		732.4	WMIN		
8	610.4	WMIN		1831.1	WMIN		

| \_| \_|

#### Note devices in last stage are very large !

#### Review from Last Time Stage Larger than that of all previous stages combined!



# **Digital Circuit Design**

- Hierarchical Design
- Basic Logic Gates
- Properties of Logic Families
- Characterization of CMOS Inverter
- Static CMOS Logic Gates
  - Ratio Logic
  - Propagation Delay
    - Simple analytical models
      - FI/OD
        - Logical Effort
    - Elmore Delay
    - Sizing of Gates
      - The Reference Inverter



- Optimal driving of Large Capacitive Loads
- Power Dissipation in Logic Circuits
  - Other Logic Styles
  - Array Logic
  - Ring Oscillators





(Discussed in Chapter 4 of Text but definitions are not rigorous)

#### Logical effort

From Wikipedia, the free encyclopedia (Dec 8, 2021)

The method of **logical effort**, a term coined by Ivan Sutherland and Bob Sproull in 1991, is a straightforward technique used to estimate delay in a CMOS circuit. Used properly, it can aid in selection of gates for a given function (including the number of stages necessary) and sizing gates to achieve the minimum delay possible for a circuit.



(Discussed in Chapter 4 of Text but definitions are not rigorous)

Propagation delay for equal rise/fall gates was derived to be

$$t_{\text{PROP}} = t_{\text{REF}} \sum_{k=1}^{n} \frac{F_{I(k+1)}}{OD_{k}}$$

Delay calculations with "logical effort" approach

Logical effort delay approach:

$$\mathbf{t}_{\mathsf{PROP}} = \mathbf{t}_{\mathsf{REF}} \sum_{k=1}^{n} \mathbf{f}_{k}$$

( $t_{REF}$  scaling factor not explicitly stated in W\_H textbook. As defined in W\_H,  $f_k$  is dimensionless)

where  $f_k$  is the "effort delay" of stage k

g<sub>k</sub>=logical effort

h<sub>k</sub>=electrical effort

$$\mathbf{t}_{\mathsf{PROP}} = \mathbf{t}_{\mathsf{REF}} \sum_{k=1}^{n} \mathbf{f}_{k} \qquad \mathbf{f}_{k} = \mathbf{g}_{k} \mathbf{h}_{k}$$

f<sub>k</sub> = "effort delay" of stage k

g<sub>k</sub>=logical effort

h<sub>k</sub>=electrical effort

Logic Effort is the ratio of the input capacitance of a gate to the input capacitance of an inverter that can deliver the same output current

Electrical Effort is the ratio of the gate load capacitance to the input capacitance of a gate



$$\mathbf{t}_{PROP} = \mathbf{t}_{REF} \sum_{k=1}^{n} \mathbf{f}_{k} \qquad \mathbf{f}_{k} = \mathbf{g}_{k} \mathbf{h}_{k}$$

Logic Effort (g) is the ratio of the input capacitance of a gate to the input capacitance of an inverter that can deliver the same output current

Electrical Effort (h) is the ratio of the gate load capacitance to the input capacitance of a gate



$$\mathbf{t}_{\mathsf{PROP}} = \mathbf{t}_{\mathsf{REF}} \sum_{k=1}^{n} \mathbf{f}_{k} \qquad \mathbf{f}_{k} = \mathbf{g}_{k} \mathbf{h}_{k}$$









$$\mathbf{t}_{\mathsf{PROP}} = \mathbf{t}_{\mathsf{REF}} \sum_{k=1}^{n} \mathbf{f}_{k} = \mathbf{t}_{\mathsf{REF}} \sum_{k=1}^{n} \mathbf{g}_{k} \mathbf{h}_{k} = \mathbf{t}_{\mathsf{REF}} \sum_{k=1}^{n} \frac{\mathbf{F}_{\mathsf{I}(\mathsf{k+1})}}{\mathsf{OD}_{k}}$$

- Note this expression is identical to what we have derived previously (t<sub>REF</sub> scaling factor not included in W\_H text)
- Probably more tedious to use the "Logical Effort" approach
- Extensions to asymmetric overdrive factors may not be trivial
- Extensions to include parasitics may be tedious as well
- Logical Effort is widely used throughout the industry

# **Digital Circuit Design**

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- Power Dissipation in Logic Circuits
  - Other Logic Styles
  - Array Logic
  - Ring Oscillators





- Interconnects have a distributed resistance and a distributed capacitance - Often modeled as resistance/unit length and capacitance per unit length
- These delay the propagation of the signal
- Effectively a transmission line
  - analysis is really complicated
- Can have much more complicated geometries



Can have much more complicated geometries









Even this lumped model is 4-th order and a closed-form solution is very tedious ! Can use "L" or other lumped segments as well (with small number some perform better than others) Need a quick (and reasonably good) approximation to the delay of a delay line !! Did anyone actually analyze a circuit like this in EE 201?





**T-Model** 



L-Model







Elmore delay:

$$t_{PD} = \sum_{i=1}^{n} \left( C_i \sum_{j=1}^{i} R_j \right)$$

It can be shown that this is a reasonably good approximation to the actual delay

- provided sufficient number of stages are used
- number does not need to be very large
- Numbering is critical (resistors and capacitors numbered from input to output)
- As stated, only applies to this specific structure

Elmore delay:

$$t_{PD} = \sum_{i=1}^{n} \left( C_i \sum_{j=1}^{i} R_j \right)$$

Note error in text on Page 161 of first edition of WH

![](_page_23_Figure_4.jpeg)

Not detailed definition on Page 150 of second edition of WH

![](_page_24_Figure_1.jpeg)

From Wikipedia (Dec 8 2021):

Elmore delay[1] is a simple approximation to the delay through an RC network in an electronic system. It is often used in applications such as logic synthesis, delay calculation, static timing analysis, placement and routing, since it is simple to compute (especially in tree structured networks, which are the vast majority of signal nets within ICs) and is reasonably accurate. Even where it is not accurate, it is usually faithful, in the sense that reducing the Elmore delay will almost always reduce the true delay, so it is still useful in optimization. [1] W.C. Elmore. *The Transient Analysis of Damped Linear Networks with Particular Regard to* 

Wideband Amplifiers. J. Applied Physics, vol. 19(1), 1948.

![](_page_25_Figure_2.jpeg)

#### **Extensions:**

![](_page_26_Figure_2.jpeg)

#### Lumped Network Model:

![](_page_26_Figure_4.jpeg)

Extensions:

1. Create a lumped element model

![](_page_27_Figure_3.jpeg)

2. Identify te a path from input to output

![](_page_27_Figure_5.jpeg)

3. Renumber elements along path from input to output and neglect off-path elen

![](_page_28_Figure_2.jpeg)

4. Use Elmore Delay equation for elements on this RC network

$$t_{PD} = \sum_{i=1}^{4} \left( C_i \sum_{j=1}^{i} R_j \right)$$

**Extensions:** 

![](_page_29_Figure_1.jpeg)

How is a resistive load handled?

**Example with resistive load:** 

![](_page_30_Figure_2.jpeg)

Elmore delay:

![](_page_30_Figure_4.jpeg)

where

![](_page_30_Figure_6.jpeg)

![](_page_30_Figure_7.jpeg)

![](_page_30_Figure_8.jpeg)

![](_page_30_Figure_9.jpeg)

#### With resistive load:

![](_page_31_Figure_2.jpeg)

Simple Elmore delay:

$$t_{PD} = \sum_{i=1}^{n-1} \left( C_i \sum_{j=1}^{i} R_j \right) + C_n \left( \left( \sum_{j=1}^{n} R_j \right) / / R_L \right)$$

Actually, R<sub>L</sub> affects all of the delays and a modestly better but modestly more complicated delay model is often used

![](_page_32_Figure_1.jpeg)

How are the number of stages chosen?

- For hand analysis, keep number of stages small (maybe 3 or 4 for simple delay line) if possible)
- If "faithfulness" is important, should keep the number of stages per unit length constant

![](_page_33_Picture_0.jpeg)

![](_page_34_Figure_1.jpeg)

$$PROP = \sum_{i=1}^{n} t$$

# **Digital Circuit Design**

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    - Simple analytical models
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  - Sizing of Gates
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![](_page_35_Picture_14.jpeg)

Optimal driving of Large **Capacitive Loads** 

![](_page_35_Picture_16.jpeg)

- Power Dissipation in Logic Circuits
- Other Logic Styles
- Array Logic
- **Ring Oscillators**

![](_page_35_Picture_21.jpeg)

## Power Dissipation in Logic Circuits

![](_page_36_Figure_1.jpeg)

## Assume current periodic with period T<sub>CL</sub>

$$P_{AVG,T} = \frac{1}{T_{CL}} \int_{t_1}^{t_1 + T_{CL}} V_{DD} I_{DD}(t) dt$$

# Power Dissipation in Logic Circuits Types of Power Dissipation

- Static
- Pipe
- Dynamic
- Leakage
  - Gate
  - Diffusion
  - Drain

![](_page_38_Figure_0.jpeg)

 $\checkmark$ 

If Boolean output averages H and L 50% of the time

$$P_{\text{STAT,AVG}} = \frac{P_{\text{H}} + P_{\text{L}}}{2}$$
$$P_{\text{STAT,AVG}} = \frac{V_{\text{DD}}(I_{\text{DDH}} + I_{\text{DDL}})}{2}$$

- Generally decreases with V<sub>DD</sub>
- I<sub>DDH</sub>=I<sub>DDL</sub>=0 for static CMOS gates so P<sub>STAT</sub>=0
- A major source of power dissipation in ratio logic circuits and the major reason CMOS is so widely used

![](_page_39_Figure_0.jpeg)

Due to conduction of both PUN and PDN during transitions

- Can be made small if transitions are fast
- Usually negligible in Static CMOS circuits

![](_page_40_Picture_0.jpeg)

# **Dynamic Power Dissipation**

![](_page_40_Figure_2.jpeg)

Due to charging and discharging  $C_L$  on logic transitions

 $C_{\rm L}$  dissipates no power but PUN and PDN dissipate power during charge and discharge of  $C_{\rm L}$ 

C<sub>L</sub> includes all gate input capacitances of loads and interconnect capacita

![](_page_41_Picture_0.jpeg)

VDD

# **Dynamic Power Dissipation**

![](_page_41_Figure_2.jpeg)

Energy stored in  $C_1$  after  $C_1$  is charged to  $V_{DD}$ :

$$E = \frac{1}{2}C_L V_{DD}^2$$

# **Dynamic Power Dissipation**

Energy supplied by  $V_{\text{DD}}$  and dissipated in  $R_{\text{PU}}$  when  $C_{\text{L}}$  charges

$$E_{DIS} = \frac{1}{2}C_L V_{DD}^2$$

Energy stored on  $C_L$  after L-H transition

$$E_{\text{STORE}} = \frac{1}{2}C_{\text{L}}V_{\text{DD}}^{2}$$
$$E = E_{\text{DIS}} + E_{\text{STORE}} = C_{\text{L}}V_{\text{DD}}^{2}$$

When the output transitions from H to L, energy stored on  $C_L$  is dissipated in PDN

Thus, energy from  $V_{DD}$  for one L-H: H-L output transition sequence is

$$E = C_L V_{DD}^2$$

If f is the average transition rate of the output, determine P<sub>AVG</sub>

![](_page_42_Figure_9.jpeg)

![](_page_43_Picture_0.jpeg)

# **Dynamic Power Dissipation**

Energy from  $V_{DD}$  for one L-H: H-L output transition sequence is

$$E=C_LV_{DD}^2$$

If f is the average transition rate of the output, determine  $\mathsf{P}_{\mathsf{AVG}}$ 

$$P_{AVG} = \frac{E}{T} = Ef$$

$$P_{DYN} = fC_L V_{DD}^2$$

 $A \xrightarrow{I_{DD}} R_{PU}$   $V_{C} \xrightarrow{V_{C}} F$   $A \xrightarrow{V_{C}} C_{L}$ 

VDD

If a gate has a transition duty cycle of 50% with a clock frequency of  $\rm f_{\rm CL}$ 

$$P_{DYN} = \frac{f_{CL}}{2} C_L V_{DD}^2$$

Note dependent on the square of V<sub>DD</sub> ! .... Want to make VDD small !!!

Major source of power dissipation in many static CMOS circuits for  $L_{min}$ >0.1u

![](_page_44_Figure_0.jpeg)

The clock transitions on every clock cycle (i.e. it has a transition duty cycle of 100%) Clock distribution can cause significant power dissipation

But if a gate has a transition duty cycle of 50% with a clock frequency of f<sub>CL</sub>

$$P_{DYN} = \frac{f_{CL}}{2} C_L V_{DD}^2$$

## **Power Dissipation**

![](_page_45_Figure_1.jpeg)

- All power is dissipated in pull-up and pull-down devices
- C<sub>L</sub> dissipates no power but PUN and PDN dissipate power when charging and discharging C<sub>L</sub>
- Dynamic power dissipation reduced by more (often much more) than a factor of 2 if minimum sizing strategy is used

![](_page_46_Picture_0.jpeg)

# Leakage Power Dissipation

#### - Gate

- with very thin gate oxides, some gate leakage current flows
- major concern in 60nm and smaller
   processes
- actually a type of static power dissipation

#### -Diffusion

- Leakage across a reverse-biased pn junction
- Dependent upon total diffusion area
- May actually be dominant power loss on longerchannel devices
- Actually a type of static power dissipation

#### -Drain

- channel current due to small V<sub>GS</sub>-V<sub>T</sub>
- of significant concern only with low V<sub>DD</sub>
   processes
- actually a type of static power dissipation

![](_page_46_Figure_15.jpeg)

![](_page_46_Figure_16.jpeg)

Example: Determine the dynamic power dissipation in the last stage of a 6-stage CMOS pad driver if used to drive a 10pF capacitive load if the system clock is 500MHz and the output changes with 50% of the clock transitions. Assume pad driver with OD of  $\theta$ =2.5 and V<sub>DD</sub>=3.5V

![](_page_47_Figure_1.jpeg)

Solution: (assume output changes with 50% of clock transitions)  $P_{DYN} = \frac{f_{CL}}{2} C_L V_{DD}^2 = \frac{5E8}{2} \bullet 10 \text{pF} \bullet 3.5^2 = 30.5 \text{ mW}$ 

Note this solution is independent of the OD and the process

Example: Will the CMOS pad driver actually be able to drive the 10pF load with a system clock of 500MHz as in the previous example in the 0.5u process?  $V_{DD}$  In 0.5u proc t<sub>REF</sub>=20ps,

![](_page_48_Figure_1.jpeg)

Solution – since outputs are data <del>de</del>pendent, output must be able to operate 500Mz:

$$t_{CLK} = \frac{1}{500 \text{MHz}} = 2 \text{nsec}$$

$$Fl_{load} = \frac{10 \text{ pF}}{4 \text{ fF}} = 2500$$

$$OD_6 = \theta^5 = 98$$

$$T_{PROP} = 5\theta \bullet T_{REF} + \frac{Fl_{load}}{OD_6} T_{REF} = \frac{Fl_{load}}{OD_6} = \frac{2500}{98} \cong 25$$

 $t_{prop} = 5 \cdot 2.5 \cdot 20psec + 25 \cdot 20psec = (12.5 + 25)20psec = 0.75nsec$ 

since t<sub>CLK</sub>>t<sub>PROP</sub>, this pad driver can drive the 10pF load at 500MHz

Example: Determine the dynamic power dissipation in the <u>next to the</u> <u>last stage</u> of a 6-stage CMOS pad driver if used to drive a 10pF capacitive load if clocked at 500MHz. Assume pad driver with OD of  $\theta$ =2.5 and V<sub>DD</sub>=3.5V

![](_page_49_Figure_1.jpeg)

#### Solution:

 $C_{IN} = \theta^5 C_{REF} = 2.5^5 \bullet 4fF = 390 fF$ 

 $P_{DYN} = f_{CL}C_LV_{DD}^2 = 5E8 \bullet 390 fF \bullet 3.5^2 = 2.4 mW$ 

Example: Is the 6-stage CMOS pad driver adequate to drive the 10pF capacitive load as fast as possible? Assume pad driver with OD of  $\theta$ =2.5 and V<sub>DD</sub>=3.5V

![](_page_50_Figure_1.jpeg)

**Solution:** 

$$n_{OPT} = ln\left(\frac{C_{L}}{C_{REF}}\right) = ln\left(\frac{10pF}{4fF}\right) = 7.8$$

No – an 8-stage pad driver would drive the load much faster (but is not needed If clocked at only 500MHz)

Example: Determine the power that would be required in the last stage of a CMOS pad driver to drive a 32-bit data bus off-chip if the capacitive load on each line is 2pF. Assume the clock speed is 500MHz and that each bit has an average 50% toggle rate. Assume V<sub>DD</sub>=3.5V

In 0.5u proc  $t_{REF}$ =20ps, C<sub>REF</sub>=4fF,R<sub>PDREF</sub>=2.5K

**Solution:** 

$$P_{DYN} = 32 \bullet \frac{f_{CL}}{2} C_L V_{DD}^2 = 32 \bullet \frac{5E8}{2} \bullet 2pF \bullet 3.5^2 = 196 mW$$

Note: A very large amount of power is required to take a large bus off-chip if bus has a high rate of activity.

![](_page_52_Picture_0.jpeg)

# **Stay Safe and Stay Healthy !**

## **End of Lecture 43**